Research on acoustic mechanism of anti-pulse baffles effect

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Abstract

Mathematical analysis of the first tangential mode energy scattering phenomenon at transverse oscillations on longitudinal metal ribs in cylindrical channel is presented. It has been shown that several absolutely rigid ribs on the cylindrical surface of the channel lead to reducing the first resonance maximum magnitude of flow frequency characteristic in the channel for the first tangential mode of transverse oscillations. The computational results correlated with the experiment have confirmed the correctness of a phenomenon model proposed in the work.

Introduction

The task of influence of several absolutely rigid longitudinal small-height ribs located on cylindrical surface of the channel to acoustic field in the cylindrical channel will be considered in the work (see. Fig.1). Such type devices sometimes called as pulse baffles [2] are used frequently as an aid of struggle against tangential modes of transverse oscillations in cylindrical channel at vibration combustion [1], [2].

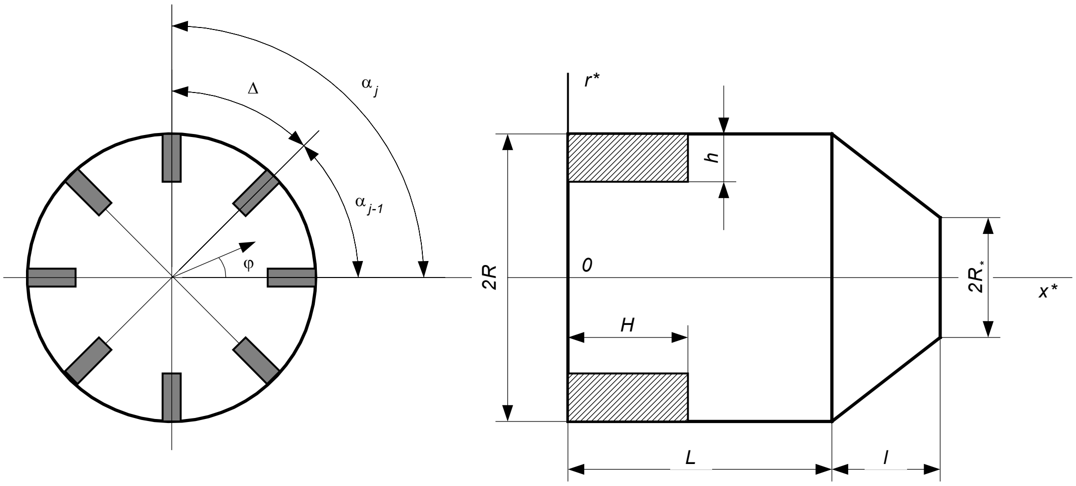

Fig.1. System of coordinates and designation

At propagation of sound wave corresponding, for instance, the first tangential mode of transverse oscillations in cylindrical channel (m=1, n=0), the phenomenon of sound scattering

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along the rib at the expense of transverse running-around the rib occurs owing to not being zero pulsation velocity vector component in this direction. Since the field of scattering wave is defined by geometrical dimensions of the ribs and the distance between them it must consist of higher modes of oscillations (m, n) than the filed of initial wave. As soon as the frequency of incident wave for higher modes of oscillations is a subcritical frequency the latter will be the waves not scattering along the length of cylinder that is quickly damped. Since energy in scattering wave is taken from energy of incident wave, the incident wave must decayed.

Gas flow in cylindrical channel with the ribs can be considered as potential flow within the accepted physical model. Notice that separation of vortexes is possible at periodic transverse running around the rib from its edges. Then they can dissipate at the expense of viscosity. Similar stabilizing effect of the baffles in great extent must depend upon level of amplitudes of pressure pulsations, so at low level of amplitudes to be considered in the given work it is possible to neglect the separation of vortexes from the edges of baffles.

1. Equation of sound field potential

Hereafter we use the cylindrical system of coordinates x^*, r^*, φ (Fig.1). Location of ribs along the circumference is uniform (with step Δ), number of ribs - N, thickness of rib is assumed as infinitely small.

We use the equation of sound field potential ψ for description of small disturbances in cylindrical channel with baffles [3].

$$\left(1-M^2\right)\frac{\partial^2\psi}{\partial x^2}-2i\sigma M\frac{\partial\psi}{\partial x}+\sigma^2\psi+\lambda^2\left[\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\psi}{\partial\varphi^2}\right]=0 \qquad (1.1)$$

This equation uses the following system of dimensionless variables:

$$M=\frac{\bar{u}}{c};\quad \sigma=\frac{\omega(L+l)}{c};\quad \lambda=\frac{(L+l)}{R};\quad x==\frac{x^*}{(L+l)};\quad r=\frac{r^*}{R} \qquad (1.2)$$

Where ω - frequency, $\bar{u}$ - stationary flow velocity, c – sound velocity in cylindrical channel.

Vector components of pulse velocity are expressed through potential ψ by dependences:

$$\delta u=\frac{u'}{\bar{u}}=-\frac{1}{M}\frac{\partial\psi}{\partial x};\quad \delta\mathrm{v}=\frac{\mathrm{v}'}{c}=-\lambda\frac{\partial\psi}{\partial r};\quad \delta\omega=\frac{\omega'}{c}=-\frac{\lambda}{r}\frac{\partial\psi}{\partial r} \qquad (1.3)$$

$$\delta\rho=\frac{\rho'}{\bar{\rho}}=i\sigma\psi+M\frac{\partial\psi}{\partial x} \qquad (1.4)$$

Considering gas as an ideal and using the condition of oscillation adiabatic character it is possible to express pressure pulsations through density pulsations:

$$\delta P=\frac{P'}{\bar{P}}=\gamma\delta\rho \quad (\gamma\text{- adiabatic index}) \qquad (1.5)$$

Since both channel walls and a rib are supposed to be absolutely rigid, sound field potential must meet the following boundary conditions: on the cylinder wall (at r=1) $\delta u=0$; on lateral surfaces of a rib $\delta\omega=0$.

We consider the conductivity α_{mn} as assigned for single mode of oscillations in the initial (x=0) and final $\left(x=\dfrac{L}{L+l}\right)$ cross-section of the channel. If a nozzle operating in supercritical regime is located in the final cross-section of the channel the acoustic conductivity of the nozzle can be determined on the procedure from work [4].

we will find potential ψ in the kind

$$\psi = Z(r,\varphi,y)\cdot exp\left(\frac{i\sigma Mx}{1-M^2}\right);\quad \text{где}\quad y=\frac{\lambda x}{\sqrt{1-M^2}} \tag{1.6}$$

Then, substituting (1.6) into (1.1), we will get

$$\frac{\partial^2 z}{\partial y^2}+\frac{\partial^2 z}{\partial r^2}+\frac{1}{r}\frac{\partial z}{\partial r}+\frac{1}{r^2}\frac{\partial^2 z}{\partial \varphi^2}+k^2 z=0 \qquad \left(k^2=\left(\frac{\sigma}{\lambda}\right)^2\frac{1}{1-M^2}\right) \tag{1.7}$$

Relation (1.7) is a Helmholtz equation for amplitude of sound field potential. it is widely used in acoustics of stationary medium [5].

2. Green function

Let us use the method of Green function for solving the equation (1.7) in such irregular area as that shown in Fig.1 [5]. we will take decision of the following non-uniform Helmholtz equation as Green function $G(r\mid r_0;\varphi\mid\varphi_0;y\mid y_0)$:

$$\frac{\partial^2 G}{\partial y^2}+\frac{\partial^2 G}{\partial r^2}+\frac{1}{r}\frac{\partial G}{\partial r}+\frac{1}{r^2}\frac{\partial^2 G}{\partial \varphi^2}+k^2 G=-4\pi\delta(y-y_0)\delta(\varphi-\varphi_0)\frac{\delta(r-r_0)}{r_0}, \tag{2.1}$$

which meets the following boundary conditions for considered cylindrical channel (without ribs)

$$\frac{\partial G}{\partial r}=0 \ \ (\text{at } r=1);\quad \frac{\partial G}{\partial y}=0 \ \ (\text{at } y=0);\quad \frac{\partial G}{\partial y}=0 \ \ (\text{at } y=y_l=\frac{\lambda L}{(L+l)\sqrt{1-M^2}}) \tag{2.2}$$

We will find the solving of the equation (2.1) in the kind of

$$G(r\mid r_0;\varphi\mid\varphi_0;y\mid y_0)=\sum_{m,n}\left[g_{mn}(y\mid y_0)\cdot J_m(\nu_{mn}r)\cdot J_m(\nu_{mn}r_0)cos(m\varphi)cos(m\varphi_0)\right] \tag{2.3}$$

where $J_m(\nu_{mn}r)$ - Bessel function and ν_{mn} - the roots of equation $J'_m(\nu_{mn}r)=0$. If now we substitute equation (2.3) into (2.1), multiply the received relation by $r_0 J_m(\nu_{mn}r)\mathbf{cos}(m\varphi)$ and to integrate the result on r_0 from 0 up to 1 and on φ_0 from 0 up to 2π, we will get

$$a_{mn}\left[\frac{\partial^2}{\partial y^2}g_{mn}(y\mid y_0)+(k^2-\nu_{mn}^2)g_{mn}(y\mid y_0)\right]=-4\pi\delta(y-y_0) \tag{2.4}$$

where

$$a_{mn} = \int_0^{2\pi}\int_0^1 J_m^2(\nu_{mn} r_0) cos^2(m\varphi_0) r_0 dr_0 d\varphi_0 .$$

Total integral of equation (2.4) meeting the boundary conditions (2.2) has of the form [5]:

$$g_{mn}(y \mid y_0) = -\frac{4\pi}{\Delta a_{mn}} \begin{cases} 4\cos(\xi y)\cos\xi(y_l - y_0),\ at\ y \le y_0 \\ 4\cos(\xi y_0)\cos\xi(y_l - y),\ at\ y \ge y_0 \end{cases} \tag{2.5}$$

where $\xi = \sqrt{k^2 - \nu_{mn}^2}$ and is an actual magnitude $\left(k^2 > \nu_{mn}^2\right)$.

In the case of complex magnitudes ξ in relation (2.5) it is necessary to transfer to hyperbolic functions.

3. Deducing an integral equation

Let us multiple equation (1.7) by G, and equation (2.1) by Z and subtract the latter from the former, then we will get

$$\begin{gathered} 4\pi\delta(y - y_0)\delta(\varphi - \varphi_0)\frac{\delta(r - r_0)}{r_0} Z(y, r, \varphi) = \\ = \left(G\frac{\partial^2 Z}{\partial y^2} - Z\frac{\partial^2 G}{\partial y^2}\right) + \frac{1}{r}\left[G\frac{\partial}{\partial r}\left(r\frac{\partial Z}{\partial r}\right) - Z\frac{\partial}{\partial r}\left(r\frac{\partial G}{\partial r}\right)\right] + \frac{1}{r^2}\left(G\frac{\partial^2 Z}{\partial \varphi^2} - Z\frac{\partial^2 G}{\partial \varphi^2}\right) \end{gathered} \tag{3.1}$$

Now we substitute in relation (3.1) y by y_0, y_0 by y, r by r_0, etc. and integrate the result in scope, then taking into account the boundary conditions for ψ and (2.2) as well as considering the main property of Green function [5]

$$G(r \mid r_0;\ \varphi \mid \varphi_0;\ y \mid y_0) = G(r_0 \mid r;\ \varphi_0 \mid \varphi;\ y_0 \mid y),$$

we will get

$$\begin{gathered} 4\pi Z(y, r, \varphi) = \int_0^{2\pi}\int_0^1 \left[\left(G\frac{\partial Z}{\partial y_0}\right)_{y_0 = y_l} - \left(G\frac{\partial Z}{\partial y_0}\right)_{y_0 = 0}\right] r_0 dr_0 d\varphi_0 - \\ - \sum_{j=1}^{N}\int_0^{y_*}\int_{1-\tau}^{1}\left[\left(Z\frac{\partial G}{\partial \varphi_0}\right)_{\varphi_0 = \alpha_j} - \left(Z\frac{\partial G}{\partial \varphi_0}\right)_{\varphi_0 = \alpha_{j-1}}\right]\frac{dr_0}{r_0} dy_0 \end{gathered} \tag{3.2}$$

In this expression: $\tau = h/R$ - dimensionless height of rib, $y_* = \lambda x_* / \sqrt{1 - M^2}$, $x_* = H/(L + l)$ - dimensionless length of rib.

Let us designate the right side of rib by sign «-», and the left side of rib by sign «+» if to move anti-clockwise the contour. Then considering continuousness of function G along φ_0, relation (3.2) can be written in the form

$$
\begin{aligned}
4\pi Z(y,r,\varphi) = & \int_0^{2\pi}\int_0^{1}\left[\left(G\frac{\partial Z}{\partial y_0}\right)_{y_0=y_l}-\left(G\frac{\partial Z}{\partial y_0}\right)_{y_0=0}\right]r_0 dr_0 d\varphi_0 + \\
& +\sum_{j=1}^{N}\int_0^{y_*}\int_{1-\tau}^{1}\left[\left(Z^+-Z^-\right)\frac{\partial G}{\partial \varphi_0}\right]_{\varphi_0=\alpha_{j-1}} dy_0\frac{dr_0}{r_0}
\end{aligned}
\tag{3.3}
$$

You can see that we succeeded in expressing the decision $Z(y,r,\varphi)$ through magnitudes of this function and its derivative on the boundary of area with the help of Green function G.

Let us expand the desired decision into a series on cylindrical functions of non-disturb task that is when the ribs are absent. Let

$$
\begin{aligned}
& Z(y,r,\varphi)=\frac{1}{4\pi}\sum_{m,n}A_{mn}(y)J_m(\nu_{mn}r)\cos m\varphi \\
& \left(\frac{\partial Z}{\partial y}\right)=\frac{1}{4\pi}\sum_{m,n}B_{mn}(y)J_m(\nu_{mn}r)\cos m\varphi
\end{aligned}
\tag{3.4}
$$

Substituting relation (3.4) and (2.3) into equation (3.3), we will get

$$
\begin{aligned}
A_{mn}(y)= & \frac{a_{mn}}{4\pi}\left[B^l_{mn}g_{mn}(y\mid y_l)-B^0_{mn}g_{mn}(y\mid 0)\right]- \\
& -\sum_{j=1}^{N} m\sin(m\alpha_{j-1})\int_0^{y_*}\int_{1-\tau}^{1}\left(Z^+-Z^-\right)_{\varphi_0=\alpha_{j-1}}\cdot J_m(\nu_{mn}r_0)g_{mn}(y\mid y_0)\frac{dr_0}{r_0}dy_0
\end{aligned}
\tag{3.5}
$$

where $B^0_{mn}=B_{mn}(0)$, $B^l_{mn}=B_{mn}(y_l)$.

Expression for potential (3.4) where coefficient $A_{mn}(y)$ is determined from integral equation (3.5), it is convenient because it allows one to evaluate the influence of ribs on every separate mode of oscillations. This influence is determined by the last member in relation (3.5). In particular, if $N=0$, or $y_*=0$, or $\tau=0$, the expression for potential of sound field in cylindrical channel without ribs follows from (3.5).

4. Determination of difference of the potential on rib surface

We draw on an approximate method for determination $\left(Z^+-Z^-\right)$ on rib surface. Near rib the equation (1.1) allows one some simplification. Since at $\tau<<1$ $\left|\dfrac{\sigma^2\psi}{\lambda^2\dfrac{\partial^2\psi}{\partial r^2}}\right|\approx\left(\tau\cdot\dfrac{\omega R}{c}\right)^2<<1$, so we can neglect member $\left(\sigma^2\psi\right)$ in equation (1.1). Then field near rib will be described by Laplace equation that is in this approximation gas near rib moves as incompressible. If we also neglect the influence of rib length on the potential of the field near rib as well as the influence of cylinder curvature in the place of rib fixation, so the task of determination of $\left(Z^+-Z^-\right)$ on rib surface reduces to the task of height-τ barrier running around on the surface by incompressible flow (see Fig.2).

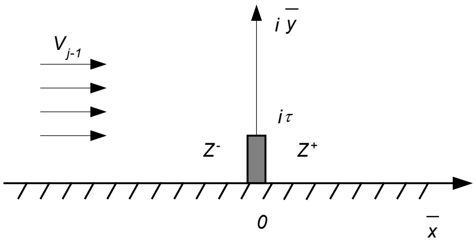

Fig.2. System of the coordinates in complex plane

Solving a similar task is possible to find by method of theory of complex variable function. Work [6] gives conformal mapping of semiplane with excluded plane segment $(\bar{x};i\bar{y})$ into semiplane of plane $(\bar{x}_1;i\bar{y}_1)$.

$$\bar{x}_1 + i\bar{y}_1 = \eta = \sqrt{\tau^2 + (\bar{x} + i\bar{y})^2} \tag{4.1}$$

But the complex potential is simple in plane $\bar{x}_1 + i\bar{y}_1 = \eta$

$$F = -V_{j-1}\eta \tag{4.2}$$

where V_{j-1} - means the specified flow velocity in infinity.

Desired potential Z must be real part of expression (4.2). So, at small $\bar{x}$ we have

$$\left(Z^+ - Z^-\right) = -2V_{j-1}\sqrt{\tau^2 - \bar{y}^2} \tag{4.3}$$

Returning to initial cylindrical system of coordinates we will get

$$\left(Z^+ - Z^-\right)_{\varphi=\alpha_{j-1}} = -2V_{j-1}\sqrt{\tau^2 - (1-r)^2} \tag{4.4}$$

We join the decision of Laplace equation with decision of wave equation (3.4) for determination of the unknown constant V_{j-1}. Since it seems impossible to reach such joining in all points of the field, that is why we will require the coincidence of velocity V_{j-1} with velocity $\left(-\frac{1}{r}\frac{\partial Z}{\partial \varphi}\right)$ only in point $r = 1$, $y = y_l$, $\varphi = \alpha_{j-1} + \frac{\Delta}{2}$, then

$$V_{j-1} = \frac{1}{4\pi}A_{mn}(y_l)m\,sin\,m\left(\alpha_{j-1} + \frac{\Delta}{2}\right)J_m(\nu_{mn}) \tag{4.5}$$

Of course, there is some arbitrariness in this operation but this way is used often [5], and as it will be seen hereafter it leads to acceptable results as the first approximation to the exact decision.

Substituting (4.5) into (4.4), and the received result into (3.5), we will find

$$A_{mn}(y) = \frac{a_{mn}}{4\pi}\left[B^l_{mn}g_{mn}(y \mid y_l) - B^0_{mn}g_{mn}(y \mid y_0) + A^l_{mn}\varepsilon\bar{\gamma}\theta_{mn}(y)\right] \tag{4.6}$$

where $\theta_{mn}(y)=\int\limits_0^{y_*} g_{mn}(y\mid y_0)dy_0$;

$$\bar{\gamma}=\sum m^2\, sin\, m\alpha_{j-1}\cdot sin\, m\left(\alpha_{j-1}+\frac{\Delta}{2}\right); \tag{4.7}$$

$$\varepsilon=\frac{2}{a_{mn}}J_m(\nu_{mn})\int\sqrt{\tau^2-(1-r)^2}\,J_m(\nu_{mn}r)\frac{dr}{r}.$$

Dependences (4.6) and (3.4) determine acoustic field in any point of cylindrical channel with ribs.

5. Frequency characteristic of the channel with ribs

Let us consider the particular case of reduced dependences, this case is related to the first tangential mode of transverse oscillations for cylindrical channel $m=1;\ n=0$. We will write expression (4.6) for two cross-sections $y=y_l$ and $y=0$:

$$\begin{aligned} A_{10}^l &= \frac{a_{10}}{4\pi}\left[B_{10}^l g_{10}(y_l\mid y_l)-B_{10}^0 g_{10}(y_l\mid 0)+A_{10}^l\varepsilon\bar{\gamma}\theta_{10}(y_l)\right] \\ A_{10}^0 &= \frac{a_{10}}{4\pi}\left[B_{10}^l g_{10}(0\mid y_l)-B_{10}^0 g_{10}(0\mid 0)+A_{10}^l\varepsilon\bar{\gamma}\theta_{10}(0)\right] \end{aligned} \tag{5.1}$$

Then we will use the boundary conditions on faces of cylinder. these conditions are written for separate mode of oscillations

$$\delta u_{10}^0=\alpha_{10}^0\delta\rho_{10}^0;\quad \delta u_{10}^l=\alpha_{10}^l\delta\rho_{10}^l \tag{5.2}$$

Here $\alpha_{10}^0, \alpha_{10}^l$ - conductivity in cross-sections $y=0$ and $y=y_l$ for the first tangential mode of transverse oscillations.

Expressing pulsations of axial components of velocity and pulsations of density by dependences (1.3), (1.4) through potential Z (1.6), we will receive

$$B_{10}^0=\beta_{10}^0A_{10}^0;\quad B_{10}^l=\beta_{10}^lA_{10}^l \tag{5.3}$$

where

$$\begin{aligned} \beta_{10}^0 &= -\frac{i\sigma M}{\lambda}\left(\frac{1}{\sqrt{1-M^2}}\right)\frac{1+\alpha_{10}^0}{1+\alpha_{10}^0M^2} \\ \beta_{10}^l &= -\frac{i\sigma M}{\lambda}\left(\frac{1}{\sqrt{1-M^2}}\right)\frac{1+\alpha_{10}^l}{1+\alpha_{10}^lM^2} \end{aligned} \tag{5.4}$$

Substituting dependences (5.3) into equations (5.1) it is possible to connect conductivity of two cross-sections $y=0$ and $y=y_l$.

$$\beta_{10}^{0} = \left\{ \left[\cos\xi y_l - \frac{\beta_{10}^{l} \sin\xi y_l}{\xi} \right] - \frac{2\chi \sin^2 \frac{1}{2}\xi y_*}{\xi^2} \right\} \times \left\{ \left[\xi \sin\xi y_l + \beta_{10}^{l} \cos\xi y_l \right] + \frac{\chi \sin\xi y_*}{\xi} \right\}^{-1} \tag{5.5}$$

Here $\chi = \varepsilon\bar{\gamma} = \dfrac{\tau^2 \nu_{10}^2 N}{4\left(\nu_{10}^2 - 1\right)} \cos\dfrac{\Delta}{2}$ (5.6)

As it has been shown in [4], resonance properties of flow in the cylindrical channel are determined by its frequency characteristic with effect on flow rate. This characteristic is a relationship of dimensionless pressure pulsations to dimensionless pulsations of flow rate in the initial cross-section $(y = 0)$:

$$m = \left[\frac{\delta P_{mn}}{\delta u_{mn} + \delta \rho_{mn}} \right]_{y=0} = -\frac{\gamma}{1 - M^2} \left[M^2 + \frac{i\sigma M}{\lambda\sqrt{1 - M^2}} \beta_{10}^{0} \right] \tag{5.7}$$

Considering the nozzle conductivity α_{10}^{0} as assigned one [4] and using the relation (5.5) it is possible to determine the frequency characteristic of cylindrical channel with the ribs for the case of the first tangential mode of transverse oscillations.

6. Results of computations and their correlation with the experiment

Fig.3 presents two computational frequency characteristics of the channel: without ribs (1) and with ribs (2). the results of acoustic experiments for the channel with ribs are shown there too (3). The experiments were conducted on the technique of work [7], satisfactory conformity between the computation (1) and the experiment for cylindrical channel without ribs have been shown in this work also.

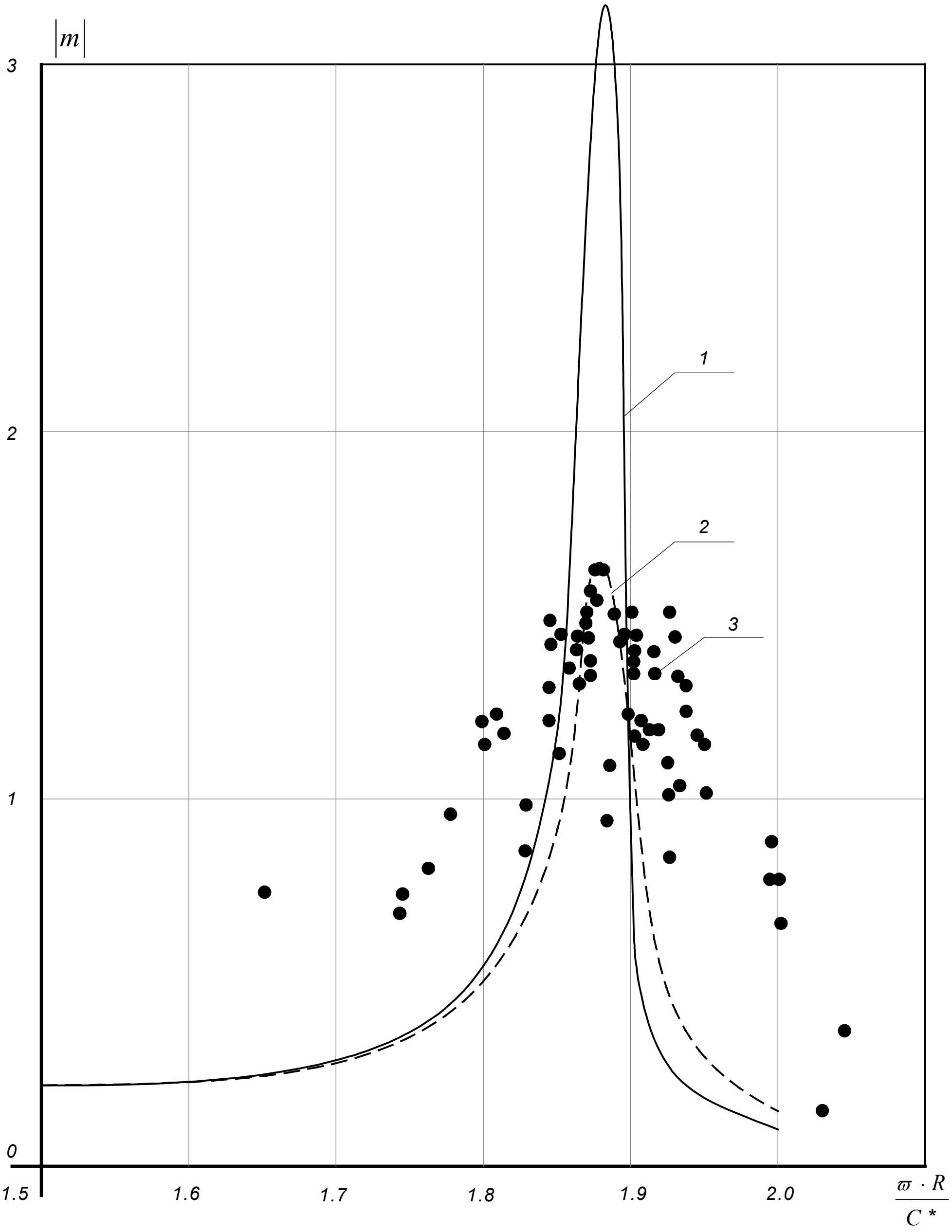

Fig.3. Frequency characteristic of the channel with ribs
(h=15 mm, H=370 mm, N=29 pcs, R=120 mm, L=490 mm, R_*=45 mm):
1 – channel without ribs (computation); 2 – channel with ribs (computation);
3 – channel with ribs (experiment).

As we can see, putting the ribs substantially decrease the magnitude of the first resonance maximum of the channel for the first tangential mode of transverse oscillations. It is observed the satisfactory conformity between the computation and experiment for the channel with ribs at least near resonance frequency. Discrepancy between the theory and the experiment far from resonance maximum has the same nature as at reading the frequency characteristic of the channel without ribs moreover that far from resonance, the experimental frequency characteristics of the channel with the ribs and without the ribs coincide between each other.

It follows from (5.5) that effectiveness of the ribs is determined by two dimensionless parameters: $\chi \sim N\left(\frac{h}{R}\right)^2$ and $y_* \sim \left(\frac{H}{L+l}\right)$. Fig.4 presents dependence of maximum value of frequency characteristic $|m|_{\max}$ from χ for different fixed magnitudes y_*. the experimental points are recorded on these curves.

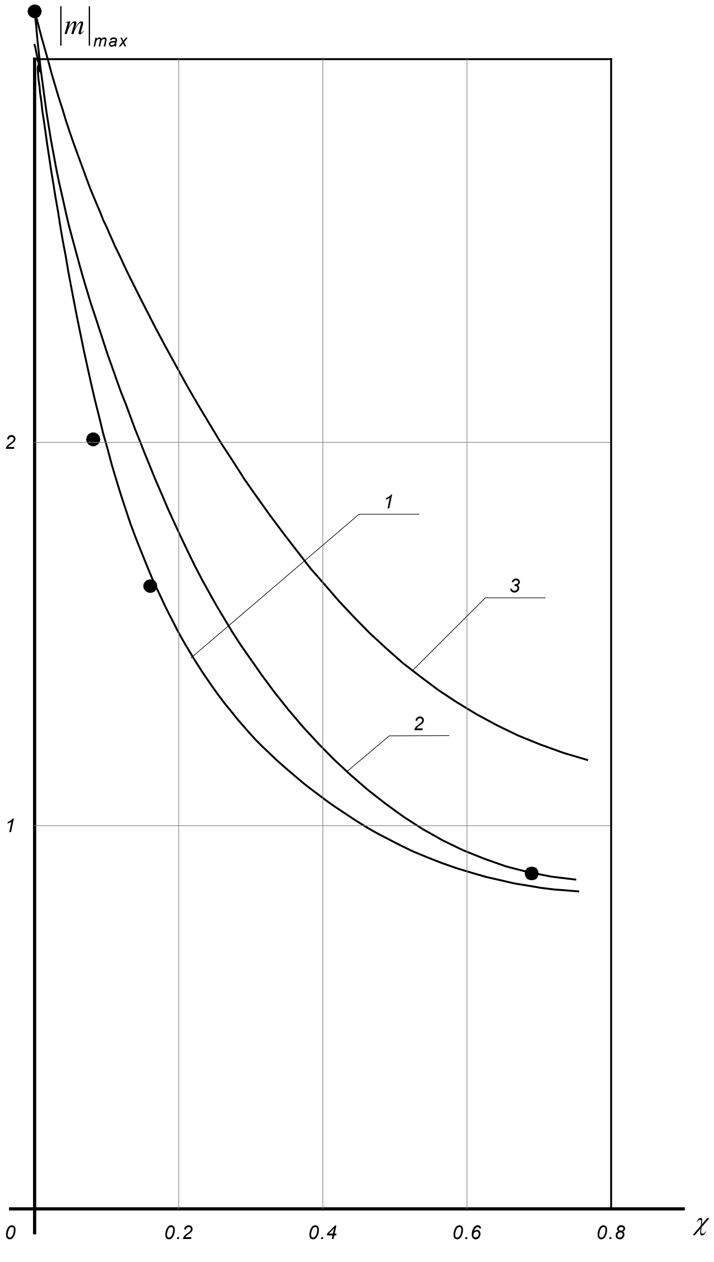

Fig.4. Dependence of magnitude of resonance maximum from frequency characteristic of the parameter (1 – y_*=3.09; 2 – y_*=2.0; 3 – y_*=1.0).

As it is shown in Fig.4, the theory and the experiment coincide with each other satisfactorily, the effectiveness of the ribs is determined by parameter χ. As it is evident from relation (5.6), the rib height effects to their effectiveness in greater extent than the length of a rib and their amount.

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